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A DIRECT STEP METHOD FOR COMPUTING WATER-SURFACE PROFILES

By Arthur A. Ezra, J. M. ASCE

HYDRAULICS DIVISION

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Highway	138, 144, 147, 148, 150, 152, 155, 163, 164, 166, 168 (Discussion: D-103, D-105, D-108, D-109, D-113, D-115, D-117, D-121, D-123, D-128)
Hydraulics	141, 143, 146, 153, 154, 159, 164, 169, 175, 178, 180 (Discussion: D-90, D-91, D-92, D-96, D-102, D-113, D-115, D-122, D-123)
Irrigation and Drainage	148, 153, 154, 156, 159, 160, 161, 162, 164, 169, 175, 178, 180 (Discussion: D-97, D-98, D-99, D-102, D-109, D-117)
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Sanitary Engineering	55, 56, 87, 91, 96, 106, 111, 118, 130, 133, 134, 135, 139, 141, 149, 153, 166, 167, 175, 176, 180 (Discussion: D-96, D-97, D-99, D-102, D-112, D-117)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115, 130, 152, 155, 157, 166, 177 (Discussion: D-86, D-103, D-108, D-109, D-115)
Structural	133, 136, 137, 142, 144, 145, 146, 147, 150, 155, 157, 158, 160, 161, 162, 163, 164, 165, 166, 168, 170, 175, 177, 179 (Discussion: D-51, D-53, D-54, D-59, D-61, D-66, D-72, D-77, D-100, D-101, D-103, D-109, D-121, D-125, D-126, D-127, D-128)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68, 121, 138, 151, 152, 172, 173 (Discussion: D-60, D-65)
Waterways	120, 123, 130, 135, 148, 154, 159, 165, 166, 167, 169 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70, D-71, D-78, D-79, D-80, D-112, D-113, D-115, D-123)

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PAPERS

A DIRECT STEP METHOD FOR COMPUTING WATER-SURFACE PROFILES

By Arthur A. Ezra,[1] J. M. ASCE

Synopsis

The direct step method is proposed to replace the computations by trial and error used in the standard step method for tracing water-surface profiles in natural channels. It is a semigraphical method applying Bernoulli's theorem and includes the effect of velocity-head changes. The effect of eddy losses, bridge-pier losses, and overbank flow can also be included when necessary. The proposed method, which is conveniently applicable in general, has been found by the writer to be especially advantageous when several water-surface profiles must be determined in the same channel.

Introduction

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for reference in the Appendix.

The proposed method seeks to eliminate the trial-and-error computations necessary in the standard step method for determining water-surface profiles in natural channels. This is accomplished by indicating the hydraulic properties of each section in a length of river channel by means of graphs, in such a fashion that, knowing the water-surface level at any section, the water-surface level at the next section (upstream or downstream) can be obtained by direct computation.

Theory

Fig. 1 represents the water-surface profile for varied flow between two sections in an open channel.

Because the total energy at section (1) is equal to the total energy at section (2) plus losses,

$$Z_1 + \frac{V^2_1}{2g} = Z_2 + \frac{V^2_2}{2g} + \text{losses} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Note.—Written comments are invited for publication; the last discussion should be submitted by September 1, 1953.

[1] Structural Engr., Corps of Engrs., San Francisco, Calif.

The losses in the reach (1)–(2) that are usually taken into account in profile computations consist of the loss of head caused by friction, the eddy losses caused by enlargement or contraction of the channel, and bend losses.

The friction loss in the reach will be computed by Manning's formula.

The eddy losses (the part of the velocity head that is not recovered) are usually taken into account in determinations of water-surface profiles as follows:

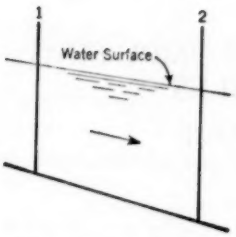

FIG. 1.—SURFACE PROFILE—VARIED FLOW IN AN OPEN CHANNEL

(*a*) They are included in the value of the roughness coefficient n in the Manning formula, and S is determined. This can be done when the value of n has been determined from observations of water-surface profiles in the same or similar reaches. The value of S in this case will represent the total rate of loss of energy head including both eddy and friction losses.

(*b*) The eddy losses are considered as a certain percentage of the change in velocity head between two sections. The total loss of head between two consecutive sections then will be the sum of the loss of head caused by friction in the reach plus the eddy losses.

Let the friction losses between sections (1) and (2) in Fig. 1 be denoted by $S_{aver}\, L$, in which S_{aver} is the average of the rates of loss of head due to friction at sections (1) and (2), and L is the length of the reach measured along the center line between sections (1) and (2).

Limiting consideration of losses to friction and eddy losses, the loss of energy head between sections (1) and (2) in Fig. 1 is

$$S_{aver}\, L + K\left(\frac{V^2_1}{2\,g} - \frac{V^2_2}{2\,g}\right) = \tfrac{1}{2}\,(S_1 + S_2)\,L + K\left(\frac{V^2_1}{2\,g} - \frac{V^2_2}{2\,g}\right)$$

in which the quantity in parentheses is the numerical difference between the larger and smaller velocity heads, irrespective of which is the larger, and S_1 and S_2 are the rates of loss of head due to friction if eddy losses are computed separately as $K\left(\frac{V^2_1}{2\,g} - \frac{V^2_2}{2\,g}\right)$. For abrupt expansions and contractions $K = 0.5$ approximately. For a gradually diverging reach K may be taken as 0.2 and for a gradually converging reach, K may be taken from 0 to 0.1, approximately.[2]

When the effect of eddy losses is included in the roughness coefficient n in Manning's formula, S_1 and S_2 (as determined from Manning's formula) represent rates of loss of energy head at sections (1) and (2).

Substituting the foregoing expression for losses in Eq. 1 yields

$$Z_1 + \frac{V^2_1}{2\,g} = Z_1 + \frac{V^2_2}{2\,g} + \tfrac{1}{2}\,(S_1 + S_2)\,L + K\left(\frac{V^2_1}{2\,g} - \frac{V^2_2}{2\,g}\right)$$

[2] U. S. Corps of Engineers Engineering Manual for Civil Works, Part CXIV, May, 1952, Chapter 9, "Computation of Backwater Curves in River Channels," Paragraph 9-06.

or

$$Z_1 + \frac{V^2_1}{2g} - \tfrac{1}{2} S_1 L = Z_2 + \frac{V^2_2}{2g} + \tfrac{1}{2} S_2 L + K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right) \ldots\ldots (2)$$

For a given discharge Q and a given roughness coefficient n, Manning's formula yields

$$Z_1 + \frac{V^2_1}{2g} - \left(\frac{L}{2}\right)\frac{n^2 V^2_1}{2.2\, R^{4/3}_1} = Z_2 + \frac{V^2_2}{2g} + \left(\frac{L}{2}\right)\frac{n^2 V^2_2}{2.2\, R^{4/3}_2} + K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right).$$

Because $V_1 = Q/A_1$ and $V_2 = Q/A_2$ it is found that

$$Z_1 + \frac{Q^2}{2g\,A^2_1} - \left(\frac{L}{2}\right)\frac{Q^2 n^2}{2.2\, A^2_1 R^{4/3}_1} = Z_2 + \frac{Q^2}{2g\,A^2_2} + \left(\frac{L}{2}\right)\frac{Q^2 n^2}{2.2\, A^2_2 R^{4/3}_2} + K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right) \ldots (3)$$

in which the term $K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right)$ is always taken as positive since the quantity in parentheses is merely the numerical difference as explained previously. Thus, for a constant discharge Q, a given roughness coefficient n, and a given distance L between sections, the expressions: $\frac{Q^2}{2g\,A^2_1} - \left(\frac{L}{2}\right)\frac{Q^2 n^2}{2.2\, A^2_1 R^{4/3}_1}$ and $\frac{Q^2}{2g\,A^2_2} + \left(\frac{L}{2}\right)\frac{Q^2 n^2}{2.2\, A^2_2 R^{4/3}_2}$ are functions of the water-surface elevation on which the values of A and R depend. The expressions of the form: $\frac{Q^2}{2g\,A^2} - \left(\frac{L}{2}\right)\frac{Q^2 n^2}{2.2\, A^2 R^{4/3}}$ may be denoted by $f(Z)$ and the expression of the form: $\frac{Q^2}{2g\,A^2} + \left(\frac{L}{2}\right)\frac{Q^2 n^2}{2.2\, A^2 R^{4/3}}$ may be denoted by $\phi(Z)$.

Eq. 3 therefore can be rewritten as

$$Z_1 + f(Z_1) = Z_2 + \phi(Z_2) + K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right) \ldots\ldots\ldots\ldots (4)$$

when Q is constant.

Description of Procedure

The direct step solution consists of the graphical application of Eq. 4. For each chosen section in the natural channel under consideration, the values of A and R for a range of water-surface elevations are determined and tabulated. This procedure is common to both the standard step solution and the proposed direct step solution. At least three water-surface elevations for each section are chosen, in the range in which the water-surface profile is expected to lie, and are used for computing the hydraulic properties.

For given values of Q and n, values of $\frac{V^2}{2g}$ and S are determined at each section for each of these elevations, and the corresponding values of $f(Z)$ and

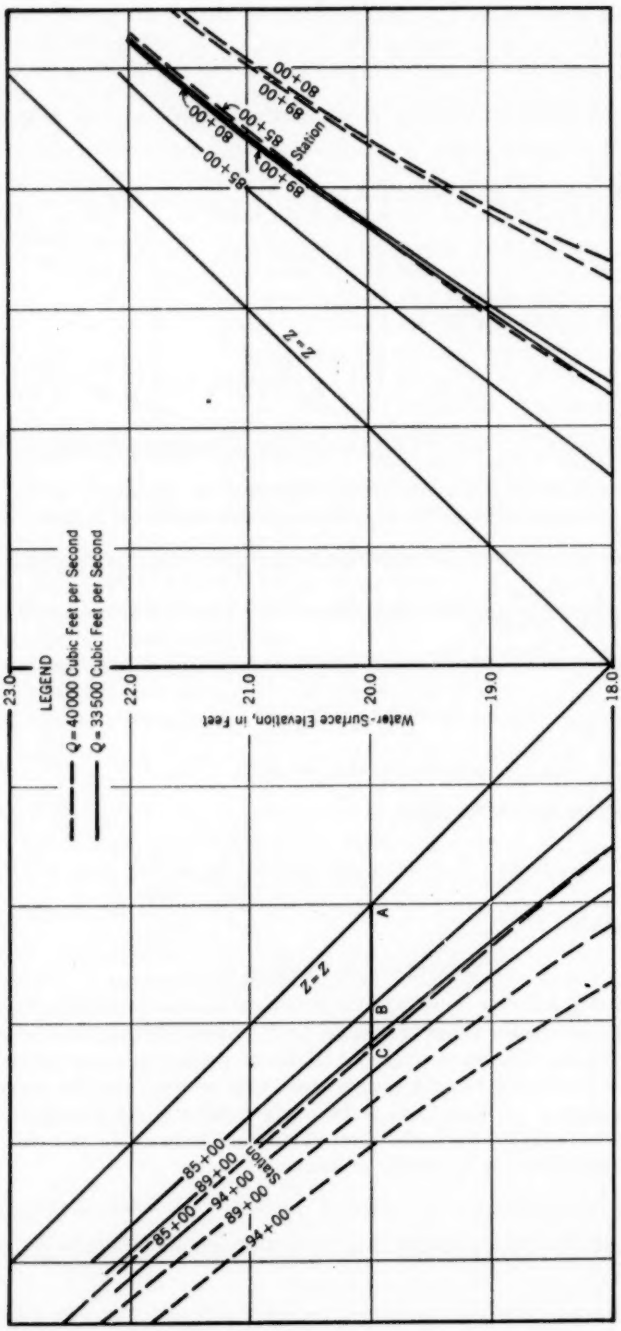

FIG. 2.—CURVES FOR THE DETERMINATION OF WATER-SURFACE PROFILES

$\phi(Z)$ are obtained. Curves of $Z + f(Z)$, $Z + \phi(Z)$, and $K\frac{V^2}{2g}$ are plotted against Z for each section, as shown in Fig. 2. For flows above critical depth, the water-surface profile is determined in upstream order. Starting with a given water-surface elevation at a section, the value of $Z + \phi(Z)$ is obtained from the appropriate curve.

Neglecting the term $K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right)$ in Eq. 4 when the effect of eddy losses is included in the value of the roughness coefficient n, it can be seen that the resulting value of $Z + \phi(Z)$ is equal to $Z + f(Z)$ for the next upstream section. Entering the curve for $Z + f(Z)$ for the next upstream section with this value, the corresponding water-surface elevation is determined directly. This procedure is repeated from section to section, tracing the desired water-surface profile. For flows below critical depth, the water-surface profile is traced downstream and the procedure is similar. Starting with the value of $Z + f(Z)$ at a section and taking this value to the $Z + \phi(Z)$ curve for the next downstream section, the corresponding water-surface elevation is determined.

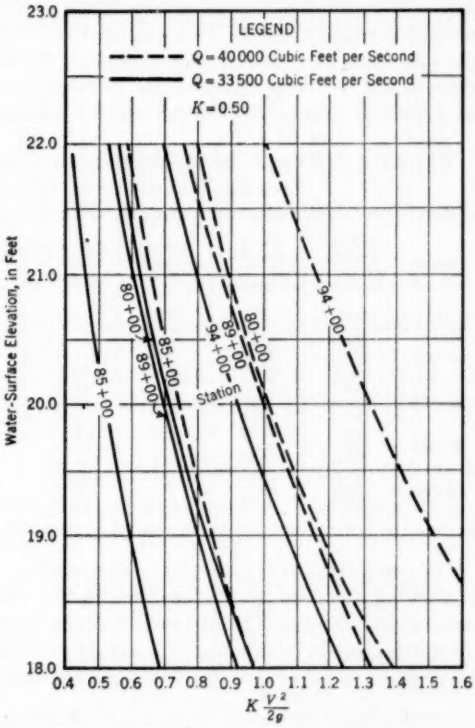

FIG. 3.—CORRECTION TO BE APPLIED TO $Z + \phi(Z)$

If it is desired to consider the eddy losses separately as a percentage of the velocity-head change between the upstream and downstream end sections of a reach, the procedure is modified. After finding the water-surface elevations for two consecutive sections when proceeding upstream (as described previously) by neglecting the term $K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right)$ in Eq. 4, the values of $K\frac{V^2}{2g}$ are found for these water-surface elevations from previously plotted curves, as shown in Fig. 3. The difference is then added to the value of $Z + \phi(Z)$ for the lower of the two sections. This corrected value is taken to the $Z + f(Z)$ curve for the higher of the two sections, and the corresponding corrected water-surface elevation is determined.

For flows below critical depth, the procedure is similar, Eq. 4 being applicable. However, the water-surface profile should be traced downstream and the correction for eddy losses $\left(\frac{K\,V^2_1}{2\,g} - \frac{K\,V^2_2}{2\,g}\right)$ should be deducted from the value of $Z + f(Z)$ for a section before this value is transferred to the $Z + \phi(Z)$ curve.

Corrections for bend losses or bridge-pier losses[3] in a reach may be made in a manner similar to that described for eddy losses.

In computing the values of $f(Z)$ used to plot the $Z + f(Z)$ curve, the value of L downstream from that section should be used. Similarly, in computing $\phi(Z)$-values for plotting the $Z + \phi(Z)$ curve, the value of L upstream from that section should be used. This insures that values of L will be common to sections at both ends of any reach, that is, the same value of L is used on both sides of Eq. 4. This choice of the L-value is important because sections taken on a natural channel usually are not equidistant from each other.

TABLE 1.—HYDRAULIC PROPERTIES OF CROSS SECTIONS, FOR VALUES OF $Q = 33{,}500$ CU FT PER SEC AND $n = 0.030$

Station	Water-surface elevation, Z, in feet	Cross-sectional area, A, in square feet	Wetted perimeter, P, in feet	Hydraulic radius, R, in feet	Mean velocity, V, in feet per second	$\frac{V^2}{2\,g}$, in feet	Gradient, S	$0.5\,\frac{V^2}{2\,g}$, in feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
80 +00	18.0	3000	238	12.60	11.17	1.94	0.00173	0.97
	20.0	3500	248	14.11	9.57	1.42	0.00110	0.71
	22.0	3970	258	15.39	8.44	1.11	0.00076	0.56
85 +00	18.0	3580	265	13.51	9.36	1.36	0.00111	0.68
	20.0	4100	273	15.02	8.17	1.04	0.00073	0.52
	22.0	4630	282	16.42	7.24	0.82	0.00051	0.41
89 +00	18.0	3080	248	12.42	10.88	1.84	0.00167	0.92
	20.0	3550	258	13.76	9.44	1.39	0.00111	0.70
	22.0	4050	268	15.11	8.27	1.06	0.00075	0.53
94 +00	18.0	2650	225	11.78	12.64	2.48	0.00243	1.24
	20.0	3100	235	13.19	10.81	1.81	0.00153	0.91
	22.0	3550	245	14.49	9.44	1.39	0.00102	0.70

In computing $f(Z)$ or $\phi(Z)$ for any section, the effect of overbank flow and of varying n can be readily included in the computations by using a weighted value of the velocity V for the sections when tabulating the hydraulic properties for the purpose of computing $f(Z)$ and $\phi(Z)$. Also, after $f(Z)$ has been obtained for a given value of $Q = Q_1$, the corresponding values of $f(Z)$ and $\phi(Z)$ for any other discharge Q_2 can be obtained by multiplying by the factor $\left(\frac{Q_2}{Q_1}\right)^2$.

The examples comprising the remainder of this paper may help to illustrate the use of the proposed direct step method.

EXAMPLE NO. 1

If a discharge of 33,500 cu ft per sec is measured in the San Lorenzo River at Santa Cruz, Calif., let it be required to determine the water-surface profile from station 80 + 00 to station 94 + 00. The water-surface elevation at

[3] "Handbook of Hydraulics," by H. W. King. McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 416.

station 80 + 00 is 18.5 ft above mean sea level for a discharge of 33,500 cu ft per sec. Assume that the value of Manning's n is 0.030, and that it includes the effect of eddy losses.

Eq. 4 is applied graphically as explained previously, neglecting the term $K\left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g}\right)$. The plotted cross sections from station 80 + 00 to station 94 + 00 have been omitted from this presentation, but the hydraulic properties

TABLE 2.—COMPUTATION OF $Z + f(Z)$ AND $Z + \phi(Z)$ FOR VALUES OF Q = 33,500 CU FT PER SEC AND n = 0.030

Station	Z, in feet	$\frac{1}{2} S L_d$, in feet	$\frac{1}{2} S L_u$, in feet	$f(Z)$, in feet	$\phi(Z)$, in feet	$Z + f(Z)$, in feet	$Z + \phi(Z)$, in feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
80+00	18.0		0.43		2.37		20.37
	20.0		0.28		1.70		21.70
	22.0		0.19		1.30		23.30
85+00	18.0	0.28	0.22	1.08	1.58	19.08	19.58
	20.0	0.18	0.15	0.86	1.19	20.86	21.19
	22.0	0.13	0.10	0.69	0.92	22.69	22.92
89+00	18.0	0.33	0.42	1.51	2.26	19.51	20.26
	20.0	0.22	0.28	1.17	1.67	21.17	21.67
	22.0	0.15	0.19	0.91	1.25	22.91	23.25
94+00	18.0	0.61		1.87		19.87	
	20.0	0.38		1.43		21.43	
	22.0	0.26		1.13		23.13	

for each section are tabulated in Table 1 for the range in which the water-surface profile is expected to lie.

The computations for $f(Z)$ and $\phi(Z)$ for three elevations are shown in Table 2. Curves of $Z + f(Z)$ and $Z + \phi(Z)$ were plotted against Z for each cross section, as shown in Fig. 2.

From Manning's formula, $S = \frac{n^2 V^2}{2.2 R^{4/3}}$; L_d was taken as the length of a reach downstream from a particular cross section; and L_u was taken as the length of a reach upstream from a particular cross section.

At station 80 + 00, for a water-surface elevation of 18.5 ft, the value of $Z + \phi(Z)$ is found from the appropriate curve in Fig. 2 to be 20.70. Taking this value to the $Z + f(Z)$ curve for the next upstream section at station 85 + 00, the corresponding water-surface elevation is found to be 19.83 ft. This procedure is repeated until station 94 + 00 is reached. The results of the profile computations have been tabulated in Table 3. This method is very advantageous in the determination of water-surface profiles in a natural channel, if several profiles, each for a different elevation at the initial section, are required.

TABLE 3.—WATER-SURFACE PROFILE, FOR A VALUE OF Q = 33,500 CU FT PER SEC

Station	$Z + f(Z)$, in feet	$Z + \phi(Z)$, in feet	Water-surface elevation, Z, in feet
(1)	(2)	(3)	(4)
80+00		20.70	18.50
85+00	20.70	21.03	19.83
89+00	21.03	21.55	19.85
94+00	21.55		20.15

Examples of this type of problem are the tracing of water-surface profiles in a tributary stream, for different stages in the main river; in a river with a tidal estuary, for different tidal elevations; or in a channel connecting two reservoirs whose elevations vary.

Example No. 2

Assume that it is required to trace the water-surface profile from station 80 + 00 to station 94 + 00 in the previous problem for a value of Q = 40,000 cu ft per sec. The roughness coefficient n in Manning's formula is equal to

TABLE 4.—Values of $Z + f(Z)$ and $Z + \phi(Z)$ for Values of Q = 40,000 Cu Ft per Sec and n = 0.030

Station (1)	Z, in feet (2)	$f(Z)$, in feet (3)	$\phi(Z)$, in feet (4)	$Z + f(Z)$, in feet (5)	$Z + \phi(Z)$, in feet (6)
80+00	18.0		3.39		21.39
	20.0		2.43		22.43
	22.0		1.86		23.86
85+00	18.0	1.54	2.26	19.54	20.26
	20.0	1.23	1.70	21.23	21.70
	22.0	0.99	1.32	22.99	23.32
89+00	18.0	2.16	3.24	20.16	21.24
	20.0	1.67	2.39	21.67	22.39
	22.0	1.30	1.79	23.30	23.79
94+00	18.0	2.67		20.67	
	20.0	2.03		22.03	
	22.0	1.62		23.62	

0.030, and the effect of eddy losses is included in the value of n. The water-surface elevation at station 80 + 00 is 19.0 ft above mean sea level for a value of Q = 40,000 cu ft per sec.

The values of $f(Z)$ and $\phi(Z)$ may be obtained for a value of Q = 40,000 cu ft per sec from the corresponding values in the previous problem, in which Q = 33,500 cu ft per sec, by multiplying them by $\left(\frac{40{,}000}{33{,}500}\right)^2 = 1.43$. The values of $f(Z)$ and $\phi(Z)$ obtained in this manner have been tabulated in Table 4, and the corresponding values of $Z + f(Z)$ and $Z + \phi(Z)$ have been plotted in Fig. 2.

TABLE 5.—Water-Surface Profile Determination for a Value of Q = 40,000 Cu Ft per Sec

Section (1)	$Z + f(Z)$, in feet (2)	$Z + \phi(Z)$, in feet (3)	Water-surface elevation, Z, in feet (4)
80+00		21.85	19.0
85+00	21.85	22.25	20.70
89+00	22.25	22.85	20.72
94+00	22.85		21.04

This also can be accomplished by a simple graphical method. In Fig. 2 straight line $Z = Z$ may be drawn along with the $Z + f(Z)$ curve and the $Z + \phi(Z)$ curve. For example, for station 85 + 00, the intercept AB between the line $Z = Z$ and the curve for $Z + f(Z)$ is equal to $Z + f(Z) - Z = f(Z)$ for Q = 33,500 cu ft per sec and a water-surface elevation of 20.00 ft. If a pair of proportional dividers is set at the ratio 1.43:1, the intercept AB may be set off at the small end of the dividers; and the intercept AC is laid off with the large end as shown—at the same water-surface elevation. The intercept AC there-

fore will be the intercept between the line $Z = Z$ and the curve for $Z + f(Z)$ for the value of $Q = 40{,}000$ cu ft per sec. Hence, C is a point on the curve of $Z + f(Z)$ for $Q = 40{,}000$ cu ft per sec. Other points on the same curve can be obtained similarly; and curves of $Z + f(Z)$ and $Z + \phi(Z)$ can be obtained for each section for $Q = 40{,}000$ cu ft per sec from the curves for $Q = 33{,}500$ cu ft per sec. The water-surface profile then may be determined from this new set of curves. The computations are tabulated in Table 5.

Example No. 3

For a discharge of 33,500 cu ft per sec in the San Lorenzo River at Santa Cruz, it is required to determine the water-surface profile from station 80 + 00 to station 94 + 00. For this discharge, the water-surface elevation at station 80 + 00 is 18.5 ft above mean sea level. The value of Manning's $n = 0.030$. Eddy losses are to be computed as 50% of the change in velocity head when the velocity head is decreasing downstream. Eddy losses may be neglected when the velocity head increases downstream.

TABLE 6.—Water-Surface Profile Determination,[a] Treating Eddy Losses Separately, for Values of $Q = 33{,}500$ Cu Ft per Sec and $K = 0.5$

Station	$Z + f(Z)$	Corrected $Z + f(Z)$	$Z + \phi(Z)$	$K\frac{V^2}{2g}$	Eddy losses	$Z + \phi(Z)$ + eddy losses	Trial Z-value	Corrected Z-value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
80 + 00			20.70	0.91			18.50	18.50
85 + 00	20.70		21.03	0.53		21.21	19.83	19.83
89 + 00	21.03	21.21	21.70	0.71	0.18	21.86	19.85	20.05
94 + 00	21.70	21.86		0.87	0.16		20.35	20.53

[a] All values are in feet.

The water-surface profile may be determined by application of Eq. 4, taking into account eddy losses. The value of K will be 0.5, and the corresponding terms are tabulated in Table 1. In Fig. 3 the curves of $K\frac{V^2}{2g}$ for $K = 0.5$ have been plotted and should be used to correct for eddy losses as explained previously. The computations for the determination of the water-surface profile are tabulated in Table 6. Starting at station 80 + 00 with a water-surface elevation of 18.50 ft (see Col. 9, Table 6), the value of $Z + \phi(Z)$ may be found from the appropriate curve in Fig. 2 to be 20.70 ft (see Col. 4, Table 6). The corresponding value of $K\frac{V^2}{2g}$ is 0.91 (see Col. 5, Table 6). Taking the value of 20.70 ft to the $Z + f(Z)$ curve for the next upstream section at station 85 + 00 (see Col. 2, Table 6), the corresponding water-surface elevation (shown in Col. 8, Table 6) is 19.83 ft, and the value of $K\frac{V^2}{2g}$ is 0.53 (see Col. 5, Table 6). Therefore, the velocity is increasing downstream and eddy losses are neglected, so that the corrected water-surface elevation is still

19.83 ft, as shown in Col. 9, Table 6. The value of $Z + \phi(Z)$ at station 85 + 00 for a water-surface elevation of 19.83 ft is found to be 21.03 ft. Going with this value to the $Z + f(Z)$ curve for the next upstream section at station 89 + 00, the water-surface elevation is found to be 19.85 ft and the corresponding value of $K\frac{V^2}{2g}$ is 0.71. Therefore, the velocity head has decreased downstream from station 89 + 00 to station 85 + 00, and the eddy losses are 0.71 ft − 0.53 ft = 0.18 ft. In order to include the effect of these eddy losses, add 0.18 ft to 21.03 ft which was previously found to be the value of $Z + \phi(Z)$ at station 85 + 00, yielding the value 21.21 ft. Going to the $Z + f(Z)$ curve for station 89 + 00 with this value, the water-surface elevation is found to be 20.05 ft.

This procedure is repeated, accounting for eddy losses when necessary, until the water-surface profile has been computed over the required distance.

APPENDIX. NOTATION

The following symbols, adopted for use in the paper and for the guidance of discussers, conform essentially with "Letter Symbols for Hydraulics" (ASA Z10.2—1942):

- A = area of the cross section, in square feet;
- g = acceleration due to gravity, in feet per second per second;
- K = ratio of eddy losses to velocity-head change between any two consecutive sections;
- L = length of a reach, in feet;
- n = roughness coefficient in the Manning formula;
- P = wetted perimeter, in feet;
- Q = discharge, in cubic feet per second;
- R = hydraulic radius in feet;
- S = rate of loss of head caused by friction at any section, as determined by the Manning formula;
- V = mean velocity of flow at any section, in feet per second; and
- Z = water-surface elevation at any section, in feet.